

134 Quiz 11 Solutions

QUESTION 1

Use the ratio test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{n}{(-7)^n}$$

For this series, $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{1}{7}$ (Enter an exact value)

Therefore, the series is convergent. (Enter "convergent" or "divergent")

$$\lim_{n \rightarrow \infty} \left| \frac{n+1}{(-7)^{n+1}} \cdot \frac{(-7)^n}{n} \right| = \lim_{n \rightarrow \infty} \left| \frac{1}{-7} \cdot \frac{n+1}{n} \right|$$

$$= \lim_{n \rightarrow \infty} \frac{1}{7} \left(1 + \frac{1}{n} \right) = \frac{1}{7} (1 + 0) = \frac{1}{7}$$

Since $\frac{1}{7} < 1$, series is convergent

QUESTION 2

Use the root test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{n} \right)^{4n^2}$$

For this series, $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = e^4$ (Enter an exact value)

Therefore, the series is divergent. (Enter "convergent" or "divergent")

$$\lim_{n \rightarrow \infty} \sqrt[n]{\left(1 + \frac{1}{n} \right)^{4n^2}} = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n} \right)^{4n} = \lim_{n \rightarrow \infty} \left[\left(1 + \frac{1}{n} \right)^n \right]^4 = e^4$$

Since $e^4 > 1$, the series is divergent

QUESTION 3

Determine whether the given series is absolutely convergent, conditionally convergent, or divergent. Prove your answer and justify each step.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n} + 3}$$

Let $b_n = \frac{1}{\sqrt{n} + 3}$. Then $\lim_{n \rightarrow \infty} b_n = 0$ and $b_{n+1} \leq b_n$. So by the alternating series test, $\sum \frac{(-1)^{n+1}}{\sqrt{n} + 3}$ converges.

Now look at $\sum \left| \frac{(-1)^{n+1}}{\sqrt{n} + 3} \right| = \sum \frac{1}{\sqrt{n} + 3}$:

We know $\sum \frac{1}{\sqrt{n}} = \sum \frac{1}{n^{1/2}}$ is a p-series with $p = \frac{1}{2}$, so it is divergent.

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{\sqrt{n} + 3}}{\frac{1}{\sqrt{n}}} = \lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\sqrt{n} + 3} = \lim_{n \rightarrow \infty} \frac{1}{1 + \frac{3}{\sqrt{n}}} = \frac{1}{1 + 0} = 1$$

So by the limit comparison test, $\sum \frac{1}{\sqrt{n} + 3}$ is divergent.

Overall conclusion: $\sum \frac{(-1)^{n+1}}{\sqrt{n} + 3}$ is conditionally convergent.

QUESTION 4

Find the radius of convergence and the interval of convergence of the given power series.

$$\sum_{n=1}^{\infty} \frac{x^n}{n^2 6^n}$$

The radius of convergence is 6. (Enter a number or enter "infinity")

The interval of convergence is . (Enter your answer using interval notation.
Use "infinity" for ∞ if needed.)

$$\begin{aligned} \lim_{n \rightarrow \infty} \left| \frac{x^{n+1}}{(n+1)^2 6^{n+1}} \cdot \frac{n^2 6^n}{x^n} \right| &= \frac{|x|}{6} \lim_{n \rightarrow \infty} \left(\frac{n}{n+1} \right)^2 = \frac{|x|}{6} \lim_{n \rightarrow \infty} \left(\frac{1}{1 + \frac{1}{n}} \right)^2 \\ &= \frac{|x|}{6} \left(\frac{1}{1+0} \right)^2 = \frac{|x|}{6} \end{aligned}$$

$$\frac{|x|}{6} < 1$$

$$|x| < 6 \quad \leftarrow \text{R.O.C.} = 6$$

$$-6 < x < 6$$

$$\text{I.O.C.} = [-6, 6]$$

Test $x = 6$

$$\sum_{n=1}^{\infty} \frac{6^n}{n^2 6^n} = \sum_{n=1}^{\infty} \frac{1}{n^2} \text{ is a p-series with } p=2$$

so it is convergent

Test $x = -6$

$$\sum_{n=1}^{\infty} \frac{(-6)^n}{n^2 6^n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \text{ converges absolutely by work above}$$

so it is convergent

QUESTION 5

Find the radius of convergence and the interval of convergence of the given power series.

$$\sum_{n=1}^{\infty} \frac{(x+1)^{2n}}{n!}$$

The radius of convergence is infinity. (Enter a number or enter "infinity")

The interval of convergence is $(-\infty, \infty)$. (Enter your answer using interval notation. Use "infinity" for ∞ if needed.)

$$\lim_{n \rightarrow \infty} \left| \frac{(x+1)^{2n+2}}{(n+1)!} \cdot \frac{n!}{(x+1)^{2n}} \right| = |x+1|^2 \lim_{n \rightarrow \infty} \frac{1}{n+1} = 0$$

$$0 < 1 \quad \text{for all } x \rightarrow \text{R.O.C.} = \infty \\ \text{I.O.C.} = (-\infty, \infty)$$